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Final Report
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RESEARCH LEADING TO PRACTICAL RUBIDIUM MASER OSCILLATORS

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WORK PERFORMED

A new laboratory has been set up and deals with problems related to optical pumping in rubidium (see Figure 1). The laboratory is equipped with two cell filling systems and soon will be equipped with an optical bench and Fabry-Perot interferometer for lamp filter and absorption cell studies.

The work performed under the contract can be divided into two parts. A theoretical investigation has shown that a limit exists in the power available out of the maser due to non-uniform illumination of the maser cell. Experimentally, a maser has been built using the cell type approach and construction of a second maser has started.

THEORY OF THE RUBIDIUM MASER

A theory describing the operation of the rubidium maser and the conditions for oscillation is summarized. It is not claimed that the theory is exact but it should describe approximately the behavior of the maser as a function of light intensity and rubidium density.

The following assumptions are made:

1) Re-radiation is quenched due to the presence of a buffer gas such as N_2.

2) The decay from the optical states takes place randomly with equal probability to any of the ground states.

3) In the ground state, in the absence of light, relaxation taking place through collisions produces an equilibrium situation with all atoms equally distributed among the eight Zeeman sublevels shown in Figure 2.

4) Illumination is uniform throughout the cell. Although in practice this situation may be hard to realize, the theory described here will give the characteristics of the maser at optimum performance.

Let us call ρ the equilibrium density matrix in the laboratory frame of reference.

$$\rho = \begin{Bmatrix} \rho_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_{33} & 0 & 0 & 0 & \rho_{37} & 0 \\ 0 & 0 & 0 & \rho_{44} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_{55} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_{66} & 0 & 0 \\ 0 & 0 & \rho_{73} & 0 & 0 & 0 & \rho_{77} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{88} \end{Bmatrix} \tag{1}$$

The hyperfine levels are numbered from high to low energy. The rate of change of any element is given by:

$$\frac{d\rho}{dt} = \left(\frac{d\rho}{dt}\right)_{OP} + \left(\frac{d\rho}{dt}\right)_{REL} + \left(\frac{d\rho}{dt}\right)_{RAD} \tag{2}$$

where:

$\left(\frac{d\rho}{dt}\right)_{OP}$ = the change with time of any elements, due to the pumping light. We will call Γ the pumping rate.

$\left(\frac{d\rho}{dt}\right)_{REL}$ = the change with time of any element due to relaxation processes in the ground states. These include collisions with buffer gas and Rb - Rb spin exchange collisions.

$\left(\frac{d\rho}{dt}\right)_{RAD}$ = the change with time due to the effect of r.f. radiation. This term is evaluated from the expression

$$\frac{d\rho_{mn}}{dt} = \frac{i}{h} \sum_k (\rho_{mk} H_{kn} - H_{mk} \rho_{kn}) \tag{3}$$

where H_{kn} and H_{mk} are the matrix elements of the interaction with the r.f. field.

From these a set of eight equations is written for the diagonal elements. One equation is sufficient for the off-diagonal element, ρ_{37}. In the setting of these equations it is assumed that only one optical hyperfine line is present in the spectrum of the lamp (i.e., an ideal Rb^{85} filter is assumed) and that the decorrelation rate due to the light is equal to $\frac{\Gamma}{2}$, the pumping rate. The set of equations will not be given here. The solution of this set of equations can be written as a power delivered by the Rb vapor:

$$P = \frac{Nh\nu}{2} \frac{x^2 \Gamma A}{(\frac{\Gamma}{2} + \gamma_2)\gamma_1 + \left(\frac{\gamma_1}{\frac{\Gamma}{2} + \gamma_2}\right)(\omega - \omega_0)^2 + (1 + \Gamma B)x^2} \quad (4)$$

where

$$A = \frac{\gamma_1(\Gamma + \gamma_1)}{5\Gamma^2 + 13\gamma_1\Gamma + 8\gamma_1^2} \quad (5)$$

$$B = \frac{3\Gamma + 4\gamma_1}{5\Gamma^2 + 13\gamma_1\Gamma + 8\gamma_1^2} \quad (6)$$

$$x = \frac{\mu_0 H_{rf}}{\hbar} \quad (7)$$

By making the field in the cavity consistent with the field produced by the atoms, we obtain a closed form solution of the operation of the maser. The solution has the following form.

$$\frac{P}{P_m} = \frac{\Gamma'}{\Gamma_m'} \left[\frac{(\Gamma' + 1)}{(2\Gamma'^2 + 9\Gamma' + 8)} - \frac{(\Gamma'/2 + r)(5\Gamma'^2 + 13\Gamma' + 8)}{(2\Gamma'^2 + 9\Gamma' + 8)} \right] \quad (8)$$

$$\Gamma_m' = \frac{\gamma_1 \hbar}{4 n \pi Q_\ell \eta \mu_0^2} = \frac{\Gamma_m}{\gamma_1}, \quad \left(\Gamma' = \frac{\Gamma}{\gamma_1}\right) \quad (9)$$

and

$$P_m = \frac{1}{2} N h \nu \Gamma_m \quad (10)$$

$$r = \frac{\gamma_2}{\gamma_1} \quad (11)$$

The various parameters are defined as follows: n is the Rb density; Q_ℓ is the cavity quality factor; η is the filling factor; μ_0 is the Bohr magneton; N is the total number of atoms; ν is the resonance frequency.

Γ_m can be understood as the "oscillation parameter" in a similar sense as "q" in the hydrogen maser. By requiring the power output to be positive (oscillating state), we can set a limit on Γ_m'. That limit is

$$\Gamma_m' \geq \frac{1}{\sqrt{80r} + 4 + 5r} \tag{12}$$

For $r = \frac{\gamma_2}{\gamma_1} = 1$ we find that Γ_m' must be smaller than 0.056. It is to be noted that γ_2 and γ_1 include here the effect of all relaxation processes except the effect of the light. A plot of the relative power output as a function of the light intensity for various values of the parameter Γ_m' is given in Figure 3.

The same method of calculation can be applied to the case of the Rb^{85} maser. This maser would operate in a way similar to the Rb^{87} maser except that the isotopes are interchanged in the cells. The power output is given by

$$\frac{P}{P_m} = \frac{\Gamma'}{\Gamma_m'}\left[\frac{(\Gamma'+1)}{3\Gamma'^2+13\Gamma'+12} - \frac{(\Gamma'/2+r)\,(7\Gamma'+19\Gamma'+12)}{3\Gamma'^2+13\Gamma'+12}\right] \tag{13}$$

The oscillation condition is more stringent in this case since the total number of states is greater and only two of these states are used for oscillations. In the case of the Rb^{85} maser Γ_m' must be smaller than 0.039 for $r = 1$, compared to 0.056 in the case of the Rb^{87} maser.

Although the theory summarized above does not describe the experimental conditions in the finest details, it can be used as a guide for understanding the processes involved and for optimizing the parameters that appear to be critical.

The most critical parameter in these equations appears to be Γ_m'. Actually Γ_m' is a measure of the spin exchange interaction, and it can be written

$$\Gamma_m' = \frac{(\gamma_c + n\bar{v}\sigma)\hbar}{4n\pi Q_\ell \eta \mu_o^2} \tag{14}$$

This parameter is plotted in Figure 4 as a function of the rubidium density. The case of our cavity is represented by the dotted line.

This theory neglects the important effect of non-uniform illumination because of absorption in the first layers of the cell. Consideration of this effect will very much alter the theory at the higher densities. The solution for such a problem, however, is not obtainable in closed form; a solution, nevertheless, is being worked out on a computer in collaboration with W. Happer of Columbia University.

EXPERIMENTAL

As mentioned earlier, a maser has been operated successfully. The cavity used was operated in a TE_{021} mode and had the following characteristics:

Q_u = 52,500 (unloaded quality factor)
β = 0.115 (coupling coefficient)

The above figures were obtained after considerable care had been taken in the fabrication of the cavity. The first cavity had a very rough internal surface and had Q values at least 15% lower than these. After polishing, the above figures were obtained. The theoretical Q is of the order of 65,000. The figures quoted above have been obtained with one end of the cavity made of wrinklefoil as shown in Figure 5. The technique, developed in this laboratory, consists in rolling a strip of wrinklefoil and a strip of flat silverplated material on each other. After heating at about 500°C to 600°C in a hydrogen furnace, the material is fused all together and shows very rugged properties. It was not possible to see any difference in quality factor between a solid end plate and one made of wrinkled foil.

Upon loading with a quartz bulb whose walls were very close to the wall of the cavity, the Q was reduced by approximately a factor of two. Since the bulb is not in the electric field, this effect is most surprising and more work must be undertaken in order to understand this phenomenon. If the cavity Q was of the order of 45,000 when loaded with the cell, the operation of the maser would be made much easier.

The cell was made of quartz and filled with 11 Torr of nitrogen and sufficient Rb^{87} to see a deposit in a reservoir made at one end. The maser could be operated with one lamp in very low field when several field dependent and field independent transitions were superimposed. The lamp used was of the type made by Varian for their rubidium magnetometers and is shown in Figure 6. With illumination from both ends of the cavity it was possible to obtain oscillation between the field independent levels alone. The power output was between 10^{-11} and 10^{-12} watt. It is estimated that a power output of 10^{-9} watt is possible.

CONCLUSION AND PLAN FOR FUTURE WORK

The work done above has shown the feasibility of operating a cell type rubidium maser between the field independent levels. The cavity Q, however, is found to be much reduced by the presence of the cell in the cavity and it would be desirable to improve this situation.

Work in the future, when appropriate programs are defined, will be performed in areas concerned with wall relaxation rates, measurements, and methods of improving the cavity Q.

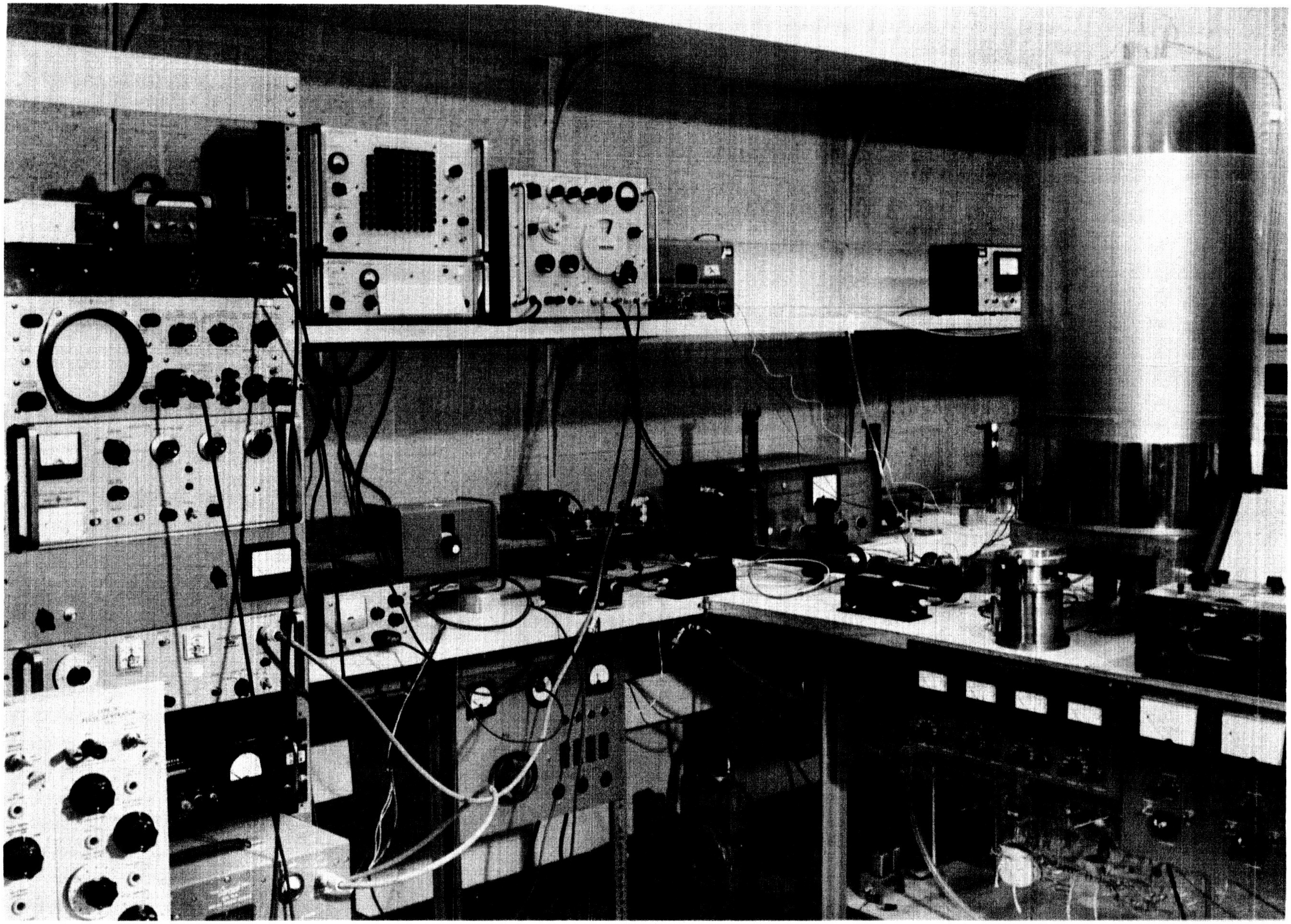


Figure 1. Rubidium Maser Laboratory.

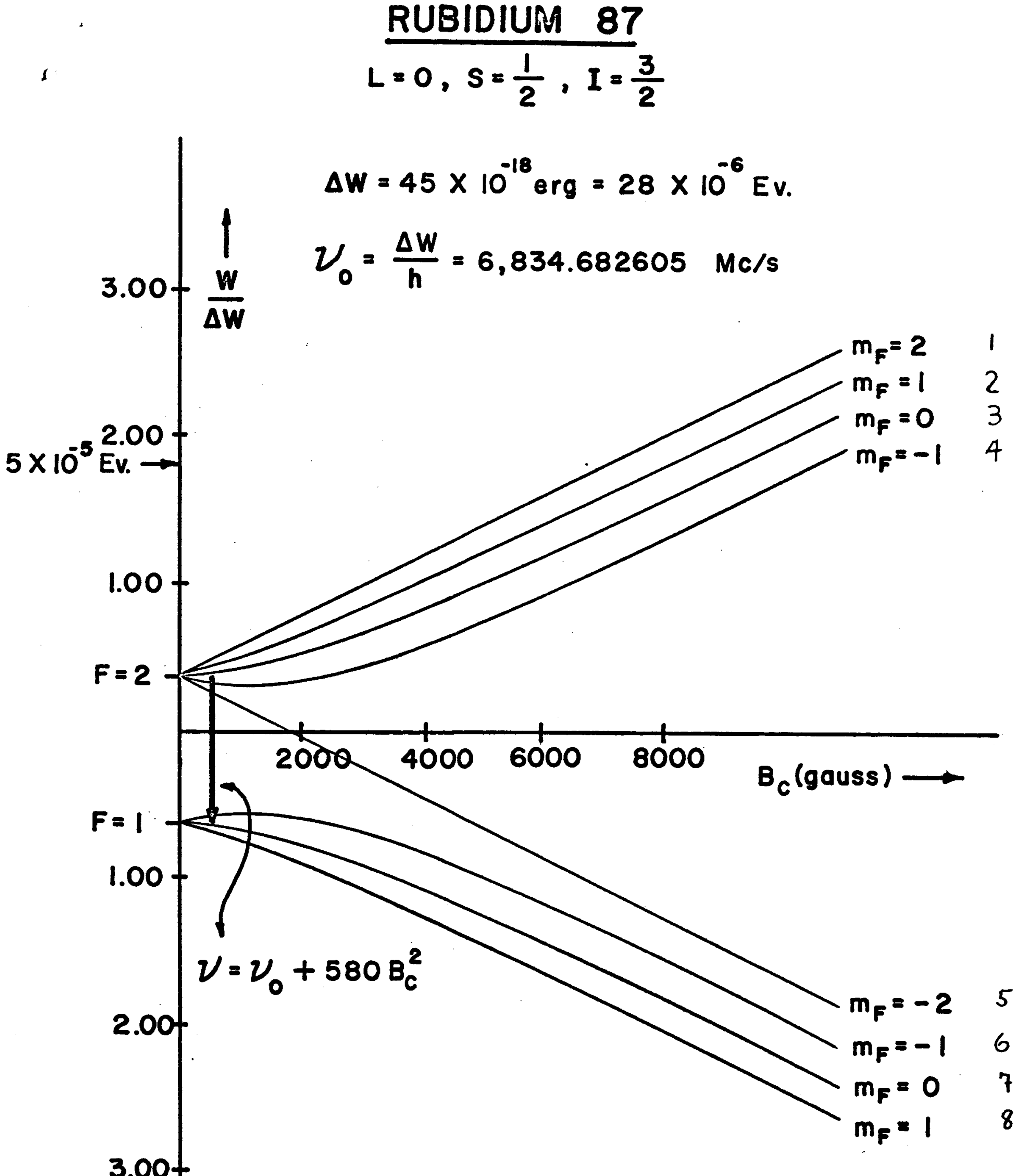
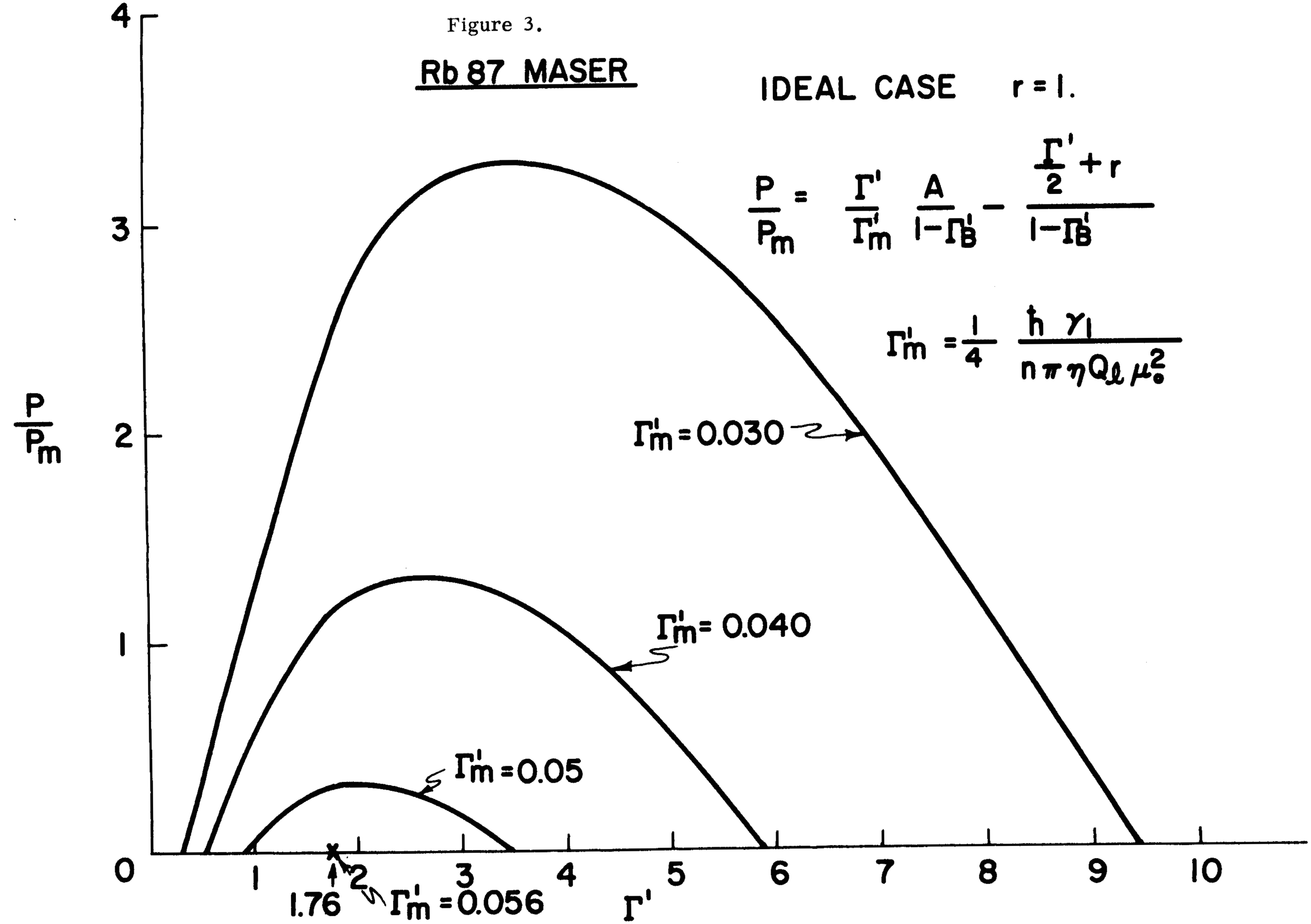
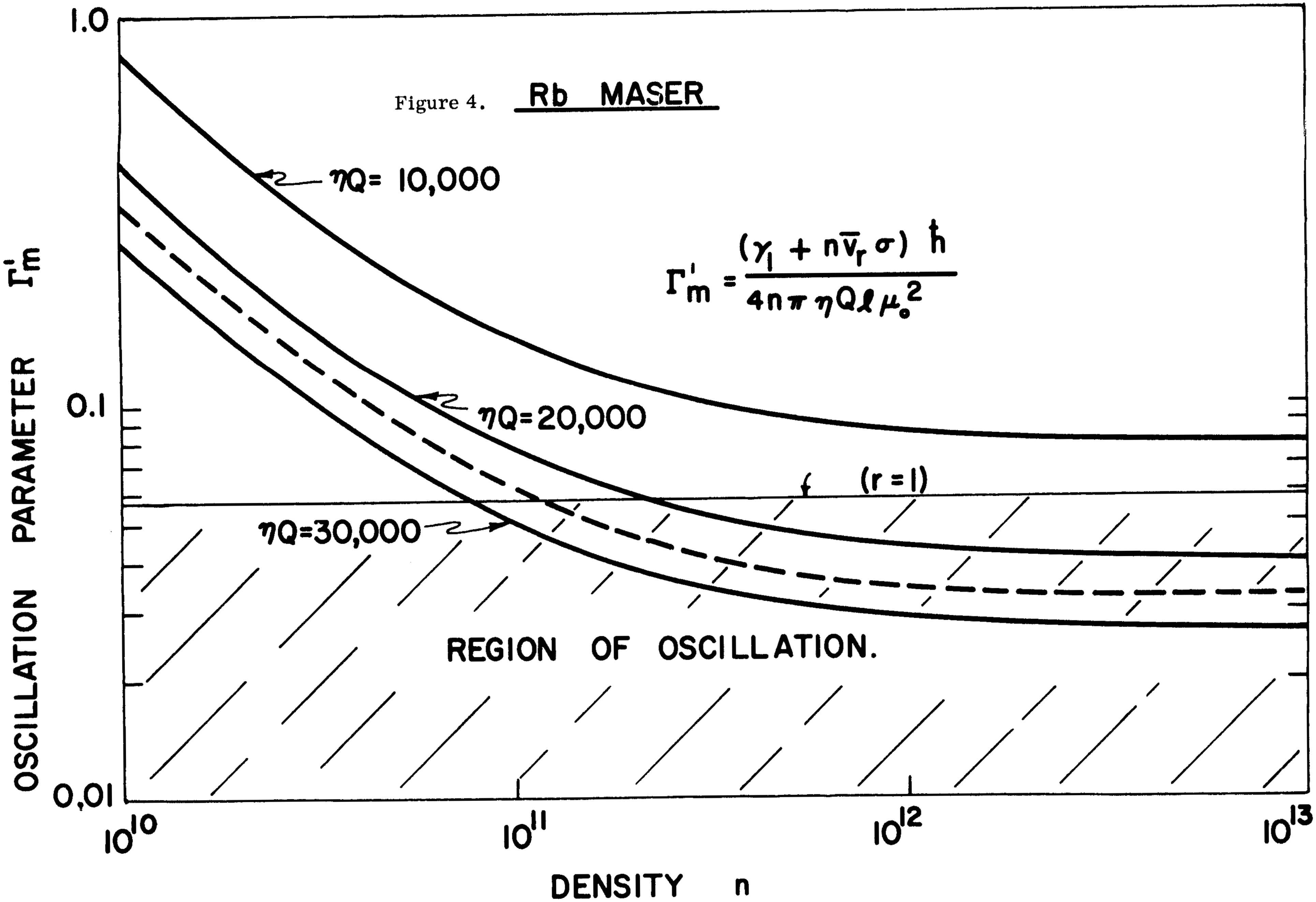


Figure 2 . Zeeman energy diagram for Rb^{87} in the ground electronic state, $5\,^2S_{\frac{1}{2}}$. The stable atomic oscillator transition connects the two states for which $F = 0$, $m_F = 0$.

Figure 3.


Rb 87 MASER
IDEAL CASE $r = 1.$
$$\frac{P}{P_m} = \frac{\Gamma'}{\Gamma'_m}\,\frac{A}{1-\Gamma'_B} - \frac{\frac{\Gamma'}{2}+r}{1-\Gamma'_B}$$
$$\Gamma'_m = \frac{1}{4}\,\frac{\hbar\,\gamma_1}{n\pi\eta Q_\ell \mu_0^2}$$
$\Gamma'_m = 0.030$
$\Gamma'_m = 0.040$
$\Gamma'_m = 0.05$
$\Gamma'_m = 0.056$
1.76
$\frac{P}{P_m}$
Γ'
0
1
2
3
4
5
6
7
8
9
10


Rb MASER
ηQ= 10,000
ηQ=20,000
ηQ=30,000
$\Gamma'_m = \frac{(\gamma_1 + n\bar{v}_r \sigma)\,\hbar}{4n\pi\,\eta Q \ell \mu_0^2}$
(r = 1)
REGION OF OSCILLATION.
1.0
0.1
0.01
10^{10}
10^{11}
10^{12}
10^{13}
OSCILLATION PARAMETER Γ'_m
DENSITY n
Figure 4.

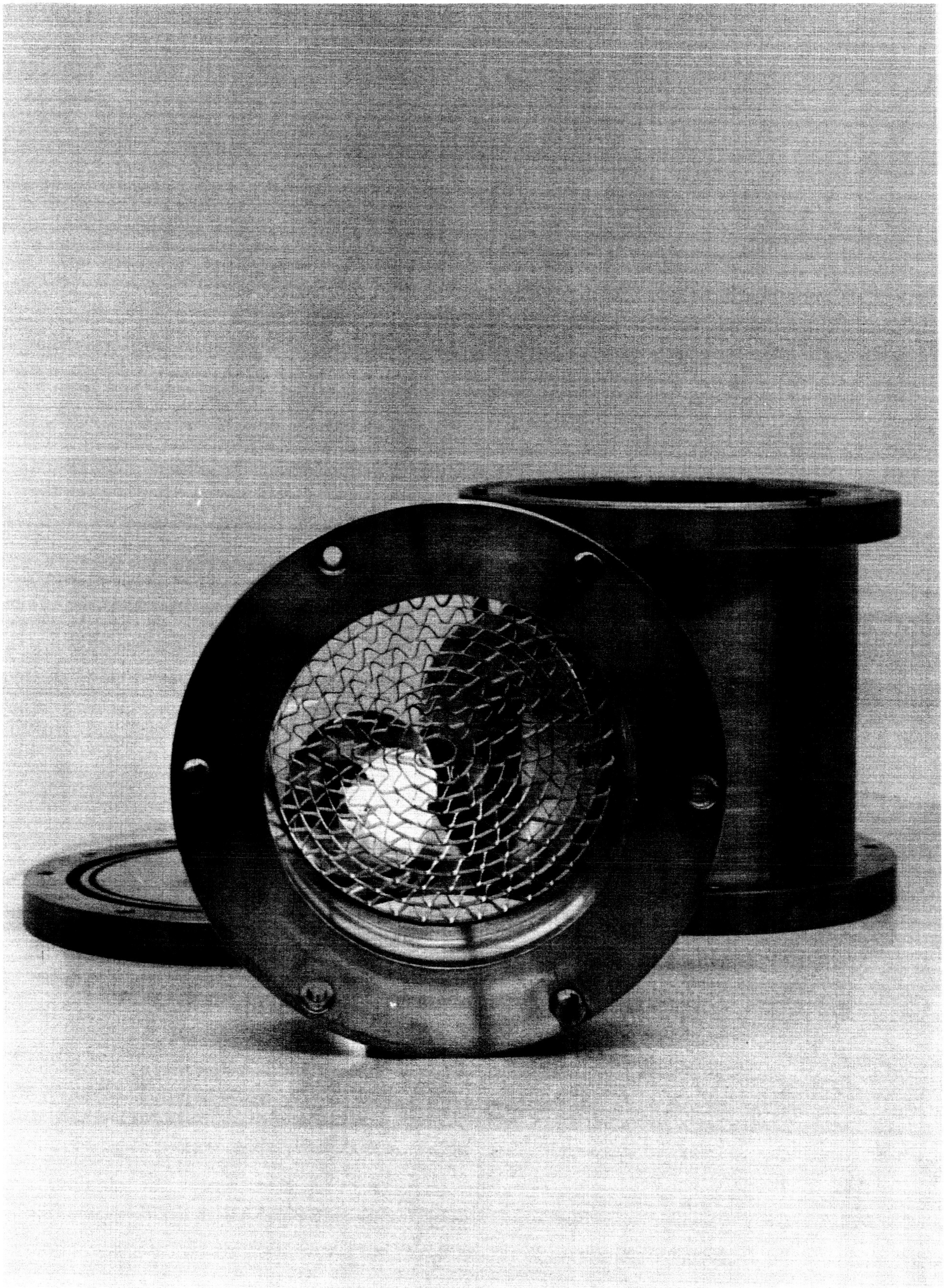


Figure 5. Maser Cavity with Transparent End Plate.

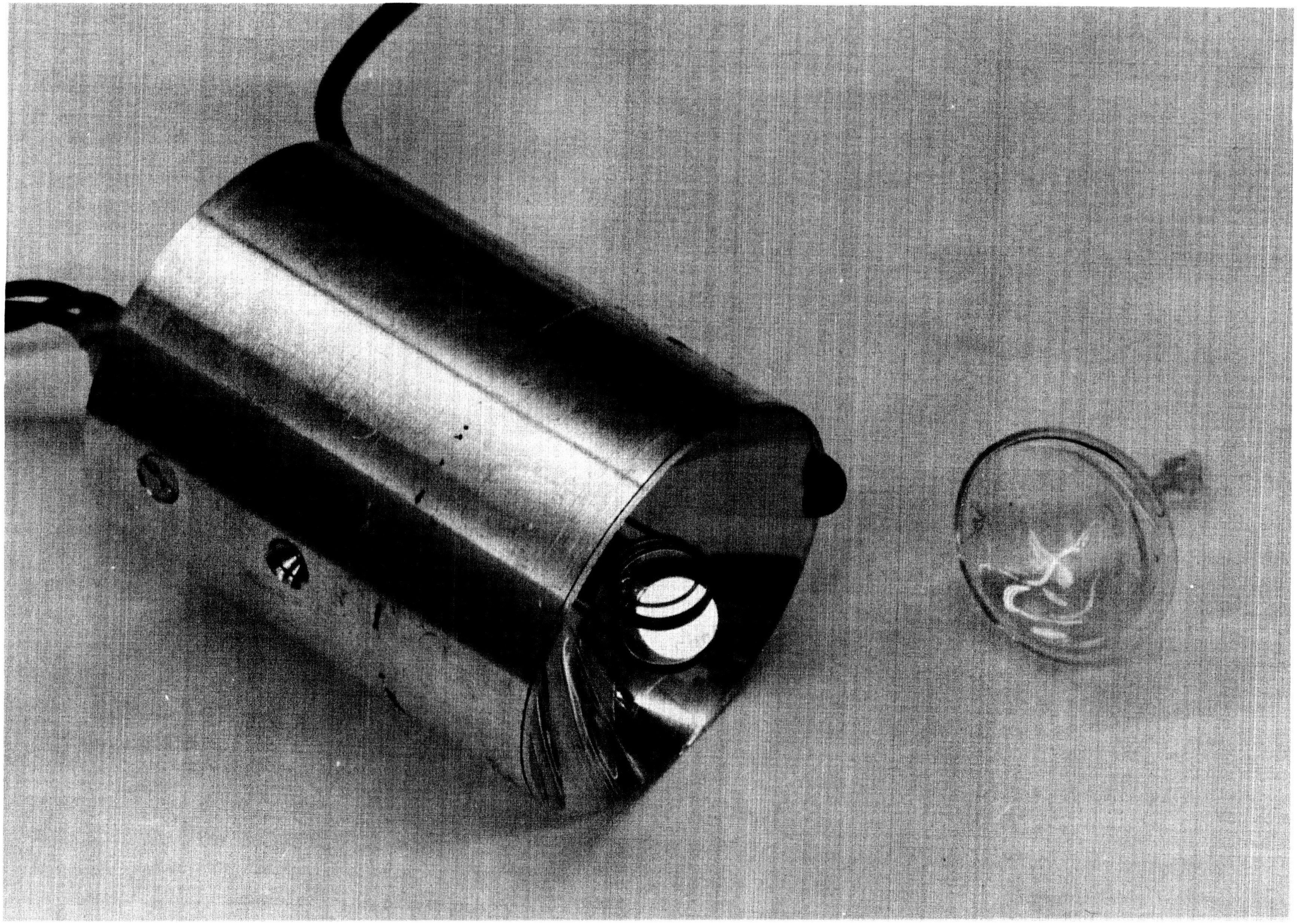


Figure 6. Varian Rb Lamp, shown with a typical pancake cell proposed as an intense lamp.